

When students are first given expressions like $3 + 4 \cdot 2$, some students think the answer is 14 and some think the answer is 11. This is why mathematicians decided on a method to simplify an expression that uses more than one operation so that everyone can agree on the answer.

There is a set of rules to follow that provides a consistent way for everyone to evaluate expressions. These rules, called the Order of Operations, must be followed in order to arrive at a correct answer. As indicated by the name, these rules state the order in which the mathematical operations are to be completed.

For additional information, see the Math Notes box in Lesson 3.1.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 5 materials.

The first step is to organize the numerical expression into parts called terms, which are single numbers or products of numbers. A numerical expression is made up of a sum or difference of terms.

Examples of numerical terms are: 4, $3(6)$, $6(9 - 4)$, $2 \cdot 3^2$, $3(5 + 2^3)$, and $\frac{16-4}{6}$.

For the problem above, $3 + 4 \cdot 2$, the terms are circled at right. $\textcircled{3} + \textcircled{4 \cdot 2}$

Each term is simplified separately, giving $3 + 8$. Then the terms are added: $3 + 8 = 11$. Thus, $3 + 4 \cdot 2 = 11$.

Example 1

$$2 \cdot 3^2 + 3(6 - 3) + 10$$

To evaluate an expression:

- Circle each term in the expression.
- Simplify each term until it is one number by:
 - Simplifying the expressions within the parentheses.
 - Evaluating each exponential part (e.g., 3^2).
 - Multiplying and dividing from left to right.
- Finally, combine terms by adding or subtracting from left to right.

$$\begin{aligned} &\textcircled{2 \cdot 3^2} + \textcircled{3(6 - 3)} + \textcircled{10} \\ &\textcircled{2 \cdot 3^2} + \textcircled{3(3)} + \textcircled{10} \\ &\textcircled{2 \cdot 9} + \textcircled{3(3)} + \textcircled{10} \\ &\textcircled{18} + \textcircled{9} + \textcircled{10} \\ &27 + 10 \\ &37 \end{aligned}$$

Example 2

- Circle the terms.
- Simplify inside the parentheses.
- Simplify the exponents.
- Multiply and divide from left to right.

Finally, add and subtract from left to right.

$$5 - 8 \div 2^2 + 6(5 + 4) - 5^2$$

- $(5) - (8 \div 2^2) + (6(5 + 4)) - (5^2)$
- $(5) - (8 \div 2^2) + (6(9)) - (5^2)$
- $(5) - (8 \div 4) + (6(9)) - (25)$
- $5 - 2 + 54 - 25$

$$32$$

Example 3

- Circle the terms.
- Multiply and divide left to right, including exponents.

Add or subtract from left to right.

$$20 + \frac{5+7}{3} - 4^2 + 12 \div 4$$

- $(20) + (\frac{5+7}{3}) - (4^2) + (12 \div 4)$
- $20 + 4 - 16 + 3$

$$11$$

Problems

Circle the terms, then simplify each expression.

1. $5 \cdot 3 + 4$

2. $10 \div 5 + 3$

3. $2(9 - 4) \cdot 7$

4. $6(7 + 3) + 8 \div 2$

5. $15 \div 3 + 7(8 + 1) - 6$

6. $\frac{9}{3} + 5 \cdot 3^2 - 2(14 - 5)$

7. $\frac{20}{6+4} + 7 \cdot 2 \div 2$

8. $\frac{5+30}{7} + 6^2 - 18 \div 9$

9. $2^3 + 8 - 16 \div 8 \cdot 2$

10. $25 - 5^2 + 9 - 3^2$

11. $5(17 - 7) + 4 \cdot 3 - 8$

12. $(5 - 2)^2 + (9 + 1)^2$

13. $4^2 + 9(2) \div 6 + (6 - 1)^2$

14. $\frac{18}{3^2} + \frac{5 \cdot 3}{5}$

15. $3(7 - 2)^2 + 8 \div 4 - 6 \cdot 5$

16. $14 \div 2 + 6 \cdot 8 \div 2 - (9 - 3)^2$

17. $\frac{27}{3} + 18 - 9 \div 3 - (3 + 4)^2$

18. $26 \cdot 2 \div 4 - (6 + 4)^2 + 3(5 - 2)^3$

19. $\left(\frac{42+3}{5}\right)^2 + 3^2 - (5 \cdot 2)^2$

Answers

1. 19

2. 5

3. 70

4. 64

5. 62

6. 30

7. 9

8. 39

9. 12

10. 0

11. 54

12. 109

13. 44

14. 5

15. 47

16. -5

17. -25

18. -6

19. -10

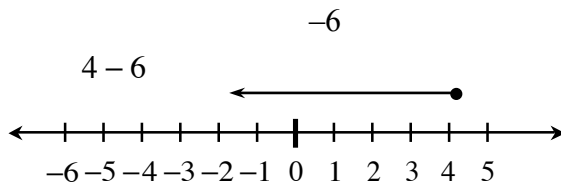
SUBTRACTION OF INTEGERS

Subtraction of integers may also be represented using the concrete models of number lines and (+) and (-) tiles. Subtraction is the opposite of addition so it makes sense to do the opposite actions of addition.

When using the number line, adding a positive integer moves to the right so subtracting a positive integer moves to the left. Adding a negative integer move to the left so subtracting a negative integer moves to the right.

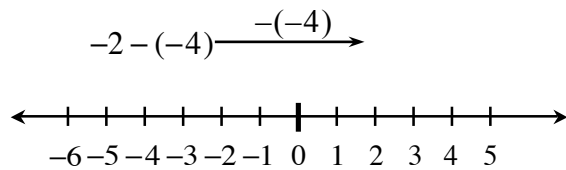
When using the tiles, addition means to place additional tile pieces into the picture and look for zeros to simplify. Subtraction means to remove tile pieces from the picture. Sometimes you will need to place zero pairs in the picture before you have a sufficient number of the desired pieces to remove. For additional information, see the Math Notes box in Lesson 3.2.2 of the *Core Connections, Course 2* text.

Example 1



$4 - 6 = -2$

Example 2



$-2 - (-4) = 2$

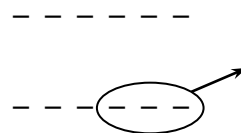
Example 3

$-6 - (-3)$

Build the first integer.

Remove three negatives.

Three negatives are left.



$-6 - (-3) = -3$

Example 4

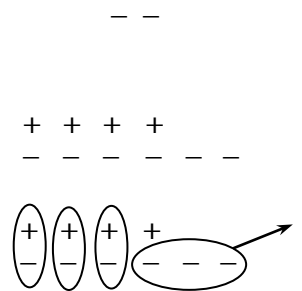
$-2 - (-3)$

Build the first integer.

It is not possible to remove three negatives so add some zeros.

Now remove three negatives and circle any zeros.

One positive remains.



$-2 - (-3) = 1$

Problems

Find each difference. Use one of the models for at least the first five differences.

1. $-6 - (-2)$

2. $2 - (-3)$

3. $6 - (-3)$

4. $3 - 7$

5. $7 - (-3)$

6. $7 - 3$

7. $5 - (3)$

8. $-12 - (-10)$

9. $-12 - 10$

10. $12 - (-10)$

11. $-6 - (-3) - 5$

12. $6 - (-3) - 5$

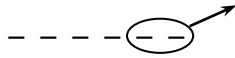
13. $8 - (-8)$

14. $-9 - 9$

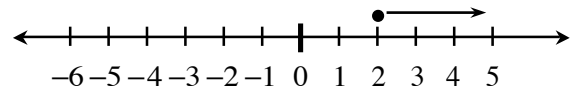
15. $-9 - 9 - (-9)$

Answers (and possible models)

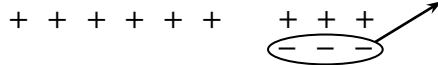
1. -4



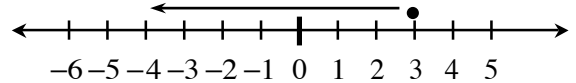
2. 5



3. 9



4. -4



5. 10

6. 4

7. 2

8. -2

9. -22

10. 22

11. -8

12. 4

13. 16

14. -18

15. -9

MULTIPLYING DECIMALS AND PERCENTS

Understanding how many places to move the decimal point in a decimal multiplication problem is connected to the multiplication of fractions and place value.

Computations involving calculating “a percent of a number” are simplified by changing the percent to a decimal.

Example 1

Multiply $(0.2) \cdot (0.3)$.

In fractions this means $\frac{2}{10} \cdot \frac{3}{10} \Rightarrow \frac{6}{100}$.

Knowing that the answer must be in the hundredths place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(tenths) = hundredths
Therefore move two places.

$$\begin{array}{r} 0.2 \\ \times 0.3 \\ \hline 0.06 \end{array}$$

Example 2

Multiply $(1.7) \cdot (0.03)$.

In fractions this means $\frac{17}{10} \cdot \frac{3}{100} \Rightarrow \frac{51}{1000}$.

Knowing that the answer must be in the thousandths place tells you how many places to move the decimal point (to the left) without using the fractions.

(tenths)(hundredths) = thousandths
Therefore move three places.

$$\begin{array}{r} 1.7 \\ \times 0.03 \\ \hline 0.051 \end{array}$$

Example 3

Calculate 17% of 32.5 without using a calculator.

Since $17\% = \frac{17}{100} = 0.17$,

17% of 32.5 $\Rightarrow (0.17) \cdot (32.5)$
 $\Rightarrow 5.525$

$$\begin{array}{r} 32.5 \\ \times 0.17 \\ \hline 2275 \\ \underline{3250} \\ 5.525 \end{array}$$

Problems

Identify the number of places to the left to move the decimal point in the product. Do not compute the product.

- | | | | | | |
|----|-----------------------|----|----------------------|----|-------------------------|
| 1. | $(0.3) \cdot (0.5)$ | 2. | $(1.5) \cdot (0.12)$ | 3. | $(1.23) \cdot (2.6)$ |
| 4. | $(0.126) \cdot (3.4)$ | 5. | $17 \cdot (32.016)$ | 6. | $(4.32) \cdot (3.1416)$ |

Compute without using a calculator.

- | | | | | | |
|-----|----------------------|-----|-----------------------|-----|-----------------------|
| 7. | $(0.8) \cdot (0.03)$ | 8. | $(3.2) \cdot (0.3)$ | 9. | $(1.75) \cdot (0.09)$ |
| 10. | $(4.5) \cdot (3.2)$ | 11. | $(1.8) \cdot (0.032)$ | 12. | $(7.89) \cdot (6.3)$ |
| 13. | 8% of 540 | 14. | 70% of 478 | 15. | 37% of 4.7 |
| 16. | 17% of 96 | 17. | 15% of 4.75 | 18. | 130% of 42 |

Answers

- | | | | | | |
|-----|-------|-----|--------|-----|--------|
| 1. | 2 | 2. | 3 | 3. | 3 |
| 4. | 4 | 5. | 3 | 6. | 6 |
| 7. | 0.024 | 8. | 0.96 | 9. | 0.1575 |
| 10. | 14.4 | 11. | 0.0576 | 12. | 49.707 |
| 13. | 43.2 | 14. | 334.6 | 15. | 1.739 |
| 16. | 16.32 | 17. | 0.7125 | 18. | 54.6 |

DIVISION BY FRACTIONS

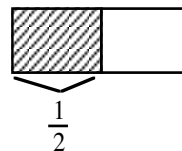
Division by fractions introduces three methods to help students understand how dividing by fractions works. In general, think of division for a problem like $8 \div 2$ as, “In 8, how many groups of 2 are there?” Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, “In $\frac{1}{2}$, how many fourths are there?”

For more information, see the Math Notes box in Lesson 3.3.1 of the *Core Connections, Course 2* text. The first two examples show how to divide fractions using a diagram.

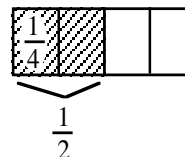
Example 1

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{4}$.

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of it.



Step 2: Then divide the *original* rectangle into four equal pieces. Each section represents $\frac{1}{4}$. In the shaded section, $\frac{1}{2}$, there are 2 fourths.



Step 3: Write the equation.

$$\frac{1}{2} \div \frac{1}{4} = 2$$

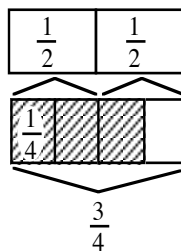
Example 2

In $\frac{3}{4}$, how many $\frac{1}{2}$'s are there?

That is, $\frac{3}{4} \div \frac{1}{2} = ?$



Start with $\frac{3}{4}$.



In $\frac{3}{4}$ there is one full $\frac{1}{2}$ shaded and half of another one (that is half of one-half).

So: $\frac{3}{4} \div \frac{1}{2} = 1 \frac{1}{2}$
(one and one-half halves)

Problems

Use the rectangular model to divide.

1. $1\frac{1}{3} \div \frac{1}{6}$ 2. $\frac{3}{2} \div \frac{3}{4}$ 3. $1 \div \frac{1}{4}$ 4. $1\frac{1}{4} \div \frac{1}{2}$ 5. $2\frac{2}{3} \div \frac{1}{9}$

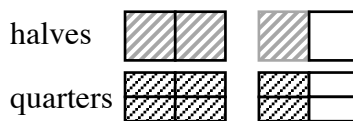
Answers

1. 8



8 sixths

2. 2



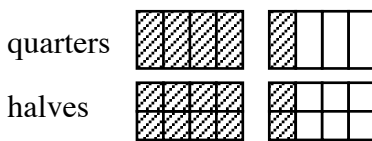
2 three fourths

3. 4



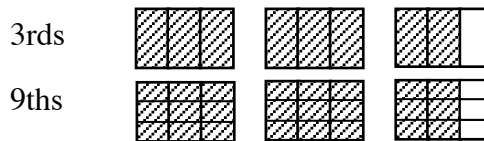
4 fourths

4. $2\frac{1}{2}$



$2\frac{1}{2}$ halves

5. 24



24 ninths

The next two examples use common denominators to divide by a fraction. Express both fractions with a common denominator, then divide the first numerator by the second.

Example 3

$$\frac{4}{5} \div \frac{2}{3} \Rightarrow \frac{12}{15} \div \frac{10}{15} \Rightarrow \frac{12}{10} \Rightarrow \frac{6}{5} \text{ or } 1\frac{1}{5}$$

Example 4

$$1\frac{1}{3} \div \frac{1}{6} \Rightarrow \frac{4}{3} \div \frac{1}{6} \Rightarrow \frac{8}{6} \div \frac{1}{6} \Rightarrow \frac{8}{1} \text{ or } 8$$

One more way to divide fractions is to use the Giant One from previous work with fractions to create a “Super Giant One.” To use a Super Giant One, write the division problem in fraction form, with a fraction in both the numerator and the denominator. Use the reciprocal of the denominator for the numerator and the denominator in the Super Giant One, multiply the fractions as usual, and simplify the resulting fraction when possible.

Example 5

$$\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{4}{1} = 4 = 2$$

Example 6

$$\frac{\frac{3}{4}}{\frac{1}{6}} = \frac{\frac{6}{1}}{\frac{6}{1}} = \frac{18}{4} = \frac{9}{2} = 4 \frac{1}{2}$$

Example 7

$$1 \frac{1}{3} \div 1 \frac{1}{2} = \frac{4}{3} \cdot \frac{2}{3} = \frac{8}{9} = \frac{8}{9}$$

Example 8

$$\frac{2}{3} \div \frac{3}{5} \Rightarrow \frac{10}{15} \div \frac{9}{15} \Rightarrow \frac{10}{9}$$

Compared to:

$$\frac{\frac{2}{3}}{\frac{3}{5}} = \frac{\frac{5}{3}}{\frac{5}{3}} = \frac{10}{9} = 1 \frac{1}{9}$$

Problems

Complete the division problems below. Use any method.

1. $\frac{3}{7} \div \frac{1}{8}$
2. $1\frac{3}{7} \div \frac{1}{2}$
3. $\frac{4}{7} \div \frac{1}{3}$
4. $1\frac{4}{7} \div \frac{1}{3}$
5. $\frac{6}{7} \div \frac{5}{8}$
6. $\frac{3}{10} \div \frac{5}{7}$
7. $2\frac{1}{3} \div \frac{5}{8}$
8. $7 \div \frac{1}{3}$
9. $1\frac{1}{3} \div \frac{2}{5}$
10. $2\frac{2}{3} \div \frac{3}{4}$
11. $3\frac{1}{3} \div \frac{5}{6}$
12. $1\frac{1}{2} \div \frac{1}{2}$
13. $\frac{5}{8} \div 1\frac{1}{4}$
14. $10\frac{1}{3} \div \frac{1}{6}$
15. $\frac{3}{5} \div 6$

Answers

1. $3\frac{3}{7}$
2. $2\frac{6}{7}$
3. $1\frac{5}{7}$
4. $4\frac{5}{7}$
5. $1\frac{13}{35}$
6. $\frac{21}{50}$
7. $3\frac{11}{15}$
8. 21
9. $3\frac{1}{3}$
10. $3\frac{5}{9}$
11. 4
12. 3
13. $\frac{1}{2}$
14. 62
15. $\frac{1}{10}$

In addition and multiplication, the order of the numbers can be reversed: $2 + 5 = 5 + 2$ and $2 \cdot 5 = 5 \cdot 2$. This is called the Commutative Property. In symbols:

The Commutative Property of Addition states: $a + b = b + a$ and

The Commutative Property of Multiplication states: $a \cdot b = b \cdot a$.

When adding three numbers or multiplying three numbers, the grouping can be changed: $(2 + 3) + 5 = 2 + (3 + 5)$ and $(2 \cdot 3) \cdot 5 = 2 \cdot (3 \cdot 5)$. This is the Associative Property. In symbols:

The Associative Property of Addition states: $(a + b) + c = a + (b + c)$ and

The Associative Property of Multiplication states: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

The Distributive Property distributes one operation over another. So far in these courses, students have seen multiplication distributed over addition. In symbols:

For all numbers a , b , and c , $a(b + c) = a \cdot b + a \cdot c$.

For example, $2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$.

For additional information see the Math Notes box in Lesson 4.1.1 of the *Core Connections, Course 2* text.

The properties of multiplication and addition allow calculations to be rearranged. Doing this is helpful when doing calculations mentally. Name the property or reason that justifies each step.

Example 1

Calculate mentally:	$4 \cdot (17 \cdot 25)$	
Step 1	$= 4 \cdot (25 \cdot 17)$	Commutative Property of Multiplication
Step 2	$= (4 \cdot 25) \cdot 17$	Associative Property of Multiplication
Step 3	$= (100) \cdot 17$	mental math
Step 4	$= 1700$	mental math

Example 2

Calculate mentally:	$8(56)$	
Step 1	$= 8(50 + 6)$	by renaming 56 as $50 + 6$
Step 2	$= 8(50) + 8(6)$	Distributive Property
Step 3	$= 400 + 48$	mental math
Step 4	$= 448$	mental math

Problems

Listed below are possible steps used to mentally calculate a problem. Give the missing reasons that justify the steps.

1. $15(29) = 15(30 + (-1))$ renamed 29 as $30 + (-1)$
 $15(30 - 1) = 15(30) + 15(-1)$ _____ a _____
 $150 + (-15) = 150 + (-10 + -5)$ renamed -15 as $-10 + (-5)$
 $150 + (-10) + (-5) = (150 + (-10)) + (-5)$ _____ b _____
 $140 + (-5) = 135$ mental math
2. $386 + 177 + 214 = 386 + 214 + 177$ _____ a _____
 $386 + 214 + 177 = (386 + 200) + 14 + 177$ _____ b _____
 $586 + 14 + 177 = (586 + 14) + 177$ _____ c _____
 $600 + 177 = 777$ mental math
3. $49(12) = 12(49)$ _____ a _____
 $12(49) = 12(50 - 1)$ renamed 49 as $50 - 1$
 $12(50 - 1) = 12(50) - 12(1)$ _____ b _____
 $12(50) - 12 = (6 \cdot 2)(50) - 12$ renamed 12 as $6 \cdot 2$
 $(6 \cdot 2)(50) - 12 = 6(2 \cdot 50) - 12$ _____ c _____
 $6(2 \cdot 50) - 12 = 6(100) - 12$ mental math
 $600 - 12 = 588$ mental math

Answers

1. a. Distributive b. Associative
2. a. Commutative b. Associative c. Associative
3. a. Commutative b. Distributive c. Associative